

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 13

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ be a polynomial function, where a_i 's are all real numbers and $a_n \neq 0$. Then the derivative function is given by

$$\frac{d f(x)}{d x} = n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \dots + 2 a_2 x + a_1$$

For functions u and v the following holds:

- $(u \pm v)' = u' \pm v'$
- $(uv)' = u'v + v'u$
- $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ provided all are defined and $v \neq 0$

Here, $u' = \frac{du}{dx}$ and $v' = \frac{dv}{dx}$

- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$

The derivative of a function f at a is defined by

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Eg: Find derivative of $f(x) = \frac{1}{x}$.

Sol: We have $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{-h}{x(x+h)} \right] = \frac{-1}{x^2}$$

- We say $\lim_{x \rightarrow a^-} f(x)$ is the expected value of f at $x=a$ given that the values of f near x to the left of a . This value is called the left hand limit of f at ' a '
- We say $\lim_{x \rightarrow a^+} f(x)$ is the expected value of f at $x=a$ given that the values of f near x to the right of a . This value is called the right hand limit of f at ' a '.
- If the right and left hand limits coincide, we call that common value as the limit of $f(x)$ at $x=a$ and denoted it by $\lim_{x \rightarrow a} f(x)$.

Eg: Find limit of function $f(x) = (x-1)^2$ at $x=1$.

Sol: For $f(x) = (x-1)^2$

Left hand limit (LHL) (at $x=1$) $= \lim_{x \rightarrow 1^-} (x-1)^2 = 0$

and Right hand limit (RHL) (at $x=1$) $= \lim_{x \rightarrow 1^+} (x-1)^2 = 0$

$\therefore$ LHL = RHL

$$\lim_{x \rightarrow 1} (x-1)^2 = 0$$

For functions f and g the following holds:

- $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}, \quad \therefore \lim_{x \rightarrow a} g(x) \neq 0$

- $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$
- $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
- $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$

